Limits & Continuity FRQ

1) a) $\lim_{x\to 1}\left(2f(x)+3g(x)\right)$

$2\lim_{x\to 1} f(x) + 3\lim_{x\to 1} g(x)$

$2(4) + 3(5) = \boxed{35}$

b) $a=-2$

I. $f(-2) = \emptyset$

$f(x)$ is not cont @ $x=-2$

$a=2$

I. $f(2)=4$

II. $\lim_{x\to 2} f(x) = 4$

III. $f(2) = \lim_{x\to 2} f(x)$

$f(x)$ is cont. @ $x=2$

$a=3$

I. $f(3) = \emptyset$

$f(x)$ is not cont @ $x=3$

c) $\lim_{x\to 0} f(f(x)) = f\left(\lim_{x\to 0} f(x)\right)$

$\lim_{x\to 0^-} f(f(x)) = \lim_{f(x)\to 1^-} f(f(x)) = 4$

$\lim_{x\to 0^+} f(f(x)) = \lim_{f(x)\to 2^+} f(f(x)) = 4$

$\lim_{x\to 0} f(f(x)) = 4$

2) a) $\lim_{t\to\infty} Y(t) = 30$

b) $Y(12) = f(12) = 20$

$\lim_{t\to 12^-} Y(t) = 20$

$\lim_{t\to 12^+} Y(t) = 20$

$Y(t)$ is cont. @ $t=12$

c) $Y(t)$ is cont on $[0,12]$

$Y(0) = 10 < 18$

$Y(12) = 20 > 18$

By IVT, $Y(t)=18$ in $(0,18)$